Multi-Robot Consensus and Coordination with Localization Uncertainty

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***Abstract*— Multi-robot coordination requires robots to share or estimate their relative positions using onboard sensors. However, the uncertainty in these position measurements is typically ignored in consensus-based protocols for rendezvous, formation control, and role assignment, which can degrade performance and safety in realistic environments. This paper presents a Wasserstein-based consensus and formation control framework that explicitly accounts for localization uncertainty by modeling each robot's position as a Gaussian belief and using the Wasserstein distance in the coordination law. This uncertainty-aware design improves robustness to sensor noise and communication constraints and guarantees exponential convergence to a consensus/formation equilibrium, with a lower bound on the convergence rate expressed in terms of uncertainty bounds and network topology. To ensure safety under uncertain localization, we further integrate a distributed chance-constrained Control Barrier Functions (CBF)–Quadratic Program (QP) layer that uses belief covariances to enforce probabilistic collision avoidance and connectivity preservation while minimally modifying the nominal controller. Extensive simulations and hardware experiments demonstrate that, for both consensus and formation tasks, the proposed framework outperforms traditional Euclidean-distance-based controllers in high-uncertainty scenarios by achieving lower steady-state error while maintaining safe inter-robot distances.**

I. Introduction

Cooperative multi-robot systems (MRS) have emerged as a vital research area due to their ability to accomplish complex tasks more efficiently, robustly, and flexibly compared to single-robot operations [1]–[3]. Numerous applications, including search and rescue, environmental monitoring, exploration, and surveillance, can benefit from deploying teams of robots that coordinate their actions and share information. Among these, rendezvous and formation control are fundamental tasks in MRS that require robust consensus-based coordination [4]. State estimation methods such as the Extended Kalman Filter (EKF) [4] are commonly used to fuse sensor and GPS measurements, providing both mean localization estimates and associated uncertainty. However, most graph-theoretic multi-robot consensus and coordination strategies rely solely on mean estimates and neglect uncertainty in decision-making [5]. This limitation becomes critical in GPS-denied or degraded environments, where GPS is unavailable or significantly degraded by noise, posing major challenges for reliable multi-robot coordination.

In the absence of GPS, relative localization, which determines the positions of robots with respect to each other or environmental landmarks, becomes crucial for achieving coordinated behavior [6]. Conventional approaches to multi-robot relative localization often rely on onboard sensors like RGB-D cameras and LiDARs [7]–[9]. These relative localization algorithms provide a mean estimate of the states along with an uncertainty, which is often overlooked.

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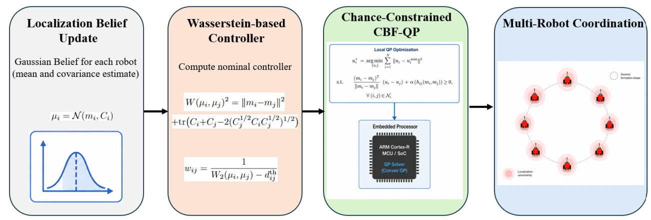


Fig. 1: Proposed uncertainty-aware multi-robot coordination framework combining Gaussian belief updates, Wasserstein-based nominal control, and chance-constrained CBF-QP safety filtering for safe formation coordination under localization uncertainty.

In MRS, uncertainty in position estimates, arising from sensor noise, signal interference, and dynamic environments, can severely degrade the performance of traditional distance-based consensus and coordination strategies, leading to instability in control and task performance [10]. To remedy this issue, we propose a novel Wasserstein-based control law that explicitly accounts for localization uncertainty in multi-robot consensus and coordination. Instead of relying on deterministic position estimates, we model each robot's location as a probability distribution, incorporating both mean position and uncertainty covariance. By leveraging the Wasserstein distance [11] as a measure of relative robot positions, the proposed approach ensures robust weight stability and mitigates the impact of uncertain localization in consensus algorithms such as rendezvous and formation control. The proposed consensus protocol improves robust and scalable multi-robot coordination in GPS-denied and adversarial environments.

Additionally, under localization uncertainty, enforcing safety based on nominal robot positions can be misleading, since the true inter-robot distances may deviate significantly from their estimates. To address this challenge, we incorporate Control Barrier Functions (CBF) [12] to enforce safety and connectivity constraints via affine control conditions embedded within a Quadratic Program (QP) optimization framework [13], thereby providing minimally invasive corrections to the nominal controller. By integrating belief covariances into this chance-constrained CBF–QP framework, the proposed approach enables probabilistic collision avoidance and connectivity preservation while maintaining the performance of the underlying consensus controllers.

The key contributions of this work are summarized as follows: *First*, we introduce a Wasserstein distance-based multi-robot protocol that models each agent's position as a

Gaussian distribution (mean and covariance) and measures inter-agent disagreement with the Wasserstein metric. This yields an *uncertainty-aware* controller for both consensus and formation control, in contrast to conventional distance-based controllers that act on position estimates. *Second*, we provide a rigorous convergence analysis proving exponential convergence of the proposed control law for both consensus and formation objectives in the presence of localization uncertainty. The analysis yields an explicit lower bound on the convergence rate as a function of the uncertainty bound and the network algebraic connectivity, linking performance to sensing quality and graph topology. *Third*, we integrate a chance-constrained CBF–QP safety filter around the Wasserstein-based consensus and formation controllers, guaranteeing probabilistic collision avoidance and connectivity preservation under localization uncertainty, and proving forward invariance of the safe set and convergence to a d_{ij}^{eff}-safe formation configuration. We validate our proposed framework on the Robotarium simulation–hardware testbed [14], demonstrating robustness to realistic sensor noise and localization uncertainty. Extensive evaluations across varying team sizes show that, for both consensus and formation tasks, our approach outperforms Euclidean-distance baselines in high-uncertainty regimes, achieving lower steady-state error while ensuring collision avoidance and connectivity through the chance-constrained CBF–QP safety layer.

II. Related Work

Effective multi-robot coordination is crucial in applications such as search and rescue, environmental monitoring, exploration, and surveillance [15]–[17]. A central tool is consensus control, where robots adjust their states to reach agreement on a common objective (e.g., rendezvous or formation) via leader–follower, distributed averaging, or fully decentralized schemes [18]–[20]. Franchi et al. [21] proposed a distributed formation control approach using range-only measurements between robots, facilitating GPS-free operation. These methods typically assume accurate relative localization so that robots can reliably exchange information and align their states. In realistic environments, however, this assumption is often violated due to sensing and communication uncertainties.

Relative localization, which estimates robot poses with respect to neighbors or landmarks, is therefore essential for enabling reliable multi-robot operation. Traditional approaches often rely on resource-intensive sensors such as RGB-D cameras and LiDAR, which require structured environments, overlapping fields of view, or map merging [7], [8]. These requirements become more challenging in dynamic or large environments, amplifying uncertainties in the pose estimates [9]. Tasooji et al. [4] combined event-triggered consensus with cooperative localization to reduce communication while maintaining coordinated motion, illustrating the potential of communication-aware designs in large-scale or remote environments with limited communication resources.

Several decentralized rendezvous strategies have also been proposed to address robustness, connectivity, and practical constraints. Park and Hutchinson [22] developed a stochastic optimal rendezvous controller robust to random node failures, while Parasuraman et al. [20] proposed a decentralized rendezvous strategy in which robots gather at dynamically assigned locations based on a hierarchical interaction graph. In [9], a weighted bearing consensus controller was introduced for coordinate-free distributed multi-robot rendezvous using wireless signal measurements to estimate relative bearings, and [15] addressed connectivity-preserving rendezvous under nonlinear dynamics and external disturbances. However, most of these works treat positions as deterministic and do not explicitly reason about localization uncertainty in the coordination layer. Zhao et al. [2] address uncertainty in a blockchain-based task allocation framework using EKF-SLAM and Gaussian Random Fields, where consensus is achieved at the task and environment level, while Ma et al. [3] study leader-following cluster consensus under measurement noise via stochastic approximation, establishing mean-square convergence for abstract agent states but still modeling positions deterministically.

To remedy this gap, we propose a Wasserstein-based consensus control law that explicitly accounts for localization uncertainty. This approach ensures robust consensus control and guarantees exponential convergence to consensus, even in the presence of uncertainty. While previous works have made significant advancements in addressing challenges in multi-robot coordination, the proposed framework distinguishes itself in the following ways: In contrast to prior works [23] that rely on accurate global positioning systems (GPS) or external motion capture systems, which may be unavailable or undesirable in certain scenarios and often introduce localization uncertainty, the proposed approach allows coordination even under a relative localization paradigm. Furthermore, while some existing studies [24] neglect the impact of sensor measurement noise on consensus-based multi-robot coordination, the present work explicitly accounts for such noise within the coordination framework, thereby improving system reliability under practical conditions.

Finally, unlike [2], [3], which handle uncertainty at the task allocation and abstract state levels via blockchain-based reward mechanisms and mean-square leader-following cluster consensus under measurement noise, our work focuses on continuous-time consensus and formation control, explicitly representing each robot's state as a Gaussian belief and incorporating the Wasserstein distance between these beliefs directly into the low-level control law.

III. Problem Formulation and Preliminaries

A. Dynamics of Robots with Localization Uncertainty

We consider a team of N robots, denoted by $V = \{1, \ldots, N\}$, each evolving in discrete time under control inputs and subject to process noise. The state of robot $i \in V$ at time step k is $p_i = [p_{x,i}, p_{y,i}, \theta_i]^T$ where $x_{i,k}$ and $y_{i,k}$ are the 2-D position coordinates and $\theta_{i,k}$ is the heading. The control input is $u_i = [v_i, \omega_i]^T$ with linear velocity $v_{i,k}$ and angular velocity $\omega_{i,k}$. The discrete-time unicycle dynamics with sampling period Δt and process noise are given by

$$
\begin{cases} x_{i,k+1} = x_{i,k} + v_{i,k}\cos(\theta_{i,k})\,\Delta t + \epsilon_{x,i,k}, \\ y_{i,k+1} = y_{i,k} + v_{i,k}\sin(\theta_{i,k})\,\Delta t + \epsilon_{y,i,k}, \\ \theta_{i,k+1} = \theta_{i,k} + \omega_{i,k}\,\Delta t + \epsilon_{\theta,i,k}, \end{cases} \tag{1}
$$

where $\epsilon_{x,i,k}, \epsilon_{y,i,k}, \epsilon_{\theta,i,k}$ represent localization errors modeled as Gaussian noise. In this work, for simplicity of analysis, we assume that only the planar position $p_{i,k} = [x_{i,k},\, y_{i,k}]^\top$ is subject to localization uncertainty, which is modeled via a Gaussian belief $\mu_{i,k} = \mathcal{N}(m_{i,k}, C_i)$, while the heading $\theta_{i,k}$ is treated as deterministic. Here, $m_{i,k} \in \mathbb{R}^2$ denotes the mean planar position of robot i, and $C_i \in \mathbb{R}^{2\times 2}$ is the covariance matrix capturing its localization uncertainty.

B. *Multi-Robot Sensing and Communication Model*

The primary assumption is that each robot is equipped with a range sensor providing position estimates subject to localization uncertainty, with measurements available within a limited sensing range R_s. We model the robots' communication network as a connected sensing graph $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{E} = \{(i,j) \mid j \in \mathcal{N}_i \Leftrightarrow \|p_i - p_j\| \le R_s\}$, with R_s representing the sensing range, and $\mathcal{N}_i$ denoting the set of neighbors of robot i in the graph. The range sensor measurements are impacted by localization errors, and the robots maintain a connected graph despite the uncertainty in their position estimates. This ensures that each robot can still obtain relevant information from its neighbors to perform coordinated tasks.

We adopt a decentralized communication model for an MRS represented by a time-varying, undirected graph $\mathcal{G}_k = \{\mathcal{V}, \mathcal{E}, A\}$, where $|\mathcal{V}| = N$ denotes the robots, $\mathcal{E}$ the communication links, and $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ the adjacency matrix, with $a_{ij} = 1$ if robots i and j are within each other's sensing range and $a_{ii} = 0$. Communication is bidirectional: if robot i can detect robot j, both robots can exchange relative position measurements z_{ij}. The graph Laplacian $\mathcal{L}$ is defined by $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. The communication topology evolves over time to reflect the dynamic nature of the robot network, influenced by both the relative positions of robots and the localization uncertainties in their measurements. Throughout the paper, we quantify the *connectivity* of the communication graph by its algebraic connectivity, i.e., the second smallest eigenvalue $\lambda_2(\mathcal{L})$ of the graph Laplacian.

IV. Wasserstein-based control law

In this section, we present a rigorous formulation of the *Wasserstein-based control law* for MRS operating under localization uncertainty. A comprehensive convergence analysis is provided, highlighting the advantages of the proposed approach over traditional distance-based control in the presence of uncertainty.

A. *Rendezvous Task*

Consider a network of N robots, where each robot i has a planar position estimate subject to localization uncertainty. We model this by a Gaussian belief $\mu_i = \mathcal{N}(m_i, C_i)$, where $m_i \in \mathbb{R}^2$ is the mean position and $C_i \in \mathbb{R}^{2\times 2}$ is the localization covariance. The rendezvous objective is to ensure that all robots meet together over time, i.e., they achieve *consensus* among their positions,

$$
\lim_{t\to\infty} p_i(t) = p_j(t), \quad \forall (i,j) \in \mathcal{E},
$$

despite the presence of localization uncertainty.

1) Traditional Distance-Based Control: A standard connectivity-preserving *distance-based consensus controller* [9] on the position estimates m_i is

$$
u_i = \dot{m}_i = -\sum_{j\in\mathcal{N}_i} w_{ij}\left(m_i - m_j\right), \tag{2}
$$

where the weights depend on the Euclidean distance between the agents:

$$
w_{ij} = \frac{1}{\|m_i - m_j\| - d_{ij}^{\mathrm{th}}}, \qquad (i,j) \in \mathcal{E}, \tag{3}
$$

with $d_{ij}^{\mathrm{th}} > 0$ is the threshold distance to maintain connectivity.

Under localization uncertainty, the controller has access only to noisy position estimates, modeled by the Gaussian beliefs $\mu_i = \mathcal{N}(m_i, C_i)$. Small fluctuations in the (noisy) relative distances $\|m_i - m_j\|$ can then drive the denominator $\|m_i - m_j\| - d_{ij}^{\mathrm{th}}$ arbitrarily close to zero, producing very large (or even singular) weights and leading to weight instability and potential divergence of the closed-loop system.

2) Proposed Wasserstein-Based Control: To explicitly account for localization uncertainty, we replace the Euclidean distance with the Wasserstein distance [11] between the Gaussian beliefs:

$$
W(\mu_i, \mu_j)^2 = \|m_i - m_j\|^2 + \mathrm{tr}\big(C_i + C_j - 2(C_j^{1/2} C_i C_j^{1/2})^{1/2}\big). \tag{4}
$$

The Wasserstein-based consensus law is

$$
u_i = \dot{m}_i = -\sum_{j\in\mathcal{N}_i} w_{ij}\,(m_i - m_j), \tag{5}
$$

with

$$
w_{ij} = \frac{1}{W_2(\mu_i, \mu_j) - d_{ij}^{\mathrm{th}}}, \qquad (i,j) \in \mathcal{E}, \tag{6}
$$

where d_{ij}^{th} is an effective separation margin. This construction smooths the dependence of the weights on both mean positions and covariances and improves robustness to localization uncertainty.

B. *Safety via Chance-Constrained Control Barrier Functions*

To enforce collision avoidance, we wrap the Wasserstein-based consensus/formation controller in a CBF–QP safety filter. Each robot maintains a Gaussian belief $\mu_i = \mathcal{N}(m_i, C_i)$ over its 2-D position, and the nominal controller produces u_i^{nom} for the mean dynamics $\dot{m}_i = u_i$. We impose the chance constraint

$$
\mathbb{P}\big(\|p_i - p_j\| \ge d_{ij}^{\mathrm{th}}\big) \ge 1 - \varepsilon, \tag{7}
$$

with $p_i \sim \mathcal{N}(m_i, C_i)$, so $p_i - p_j \sim \mathcal{N}(m_i - m_j, C_i + C_j)$. This is conservatively enforced via the inflated margin

$$
d_{ij}^{\mathrm{eff}} = d_{ij}^{\mathrm{th}} + \kappa_\varepsilon \sqrt{\lambda_{\max}(C_i + C_j)}, \tag{8}
$$

where κ_ε depends on the desired confidence. Define the pairwise barrier function

$$h_{ij}(m_i, m_j) = \|m_i - m_j\| - d_{ij}^{\text{eff}}, \tag{9}$$

and enforce the CBF condition

$$\dot{h}_{ij}(m_i, m_j, u_i, u_j) + \alpha\big(h_{ij}(m_i, m_j)\big) \geq 0, \tag{10}$$

with an extended class-$\mathcal{K}$ function $\alpha(\cdot)$. For the single-integrator means,

$$\dot{h}_{ij} = \frac{(m_i - m_j)^\top}{\|m_i - m_j\|}\,(u_i - u_j),$$

so (10) is affine in (u_i, u_j).

At each step we compute the applied inputs via

$$u_i^\star = \arg\min_{\{u_i\}} \sum_{i=1}^{N} \left\|u_i - u_i^{\text{nom}}\right\|^2 \tag{11}$$

$$\text{s.t.}\quad \frac{(m_i - m_j)^\top}{\|m_i - m_j\|}\,(u_i - u_j) + \alpha\big(h_{ij}(m_i, m_j)\big) \geq 0. \forall j \in \mathcal{N}_i$$

Here, u_i^{nom} is the nominal velocity obtained by our controller in (5). Thus, the Wasserstein controller drives the Gaussian means to distributed consensus or formation control, while the CBF–QP layer minimally adjusts the inputs to maintain a probabilistic separation margin between robots.

Assumption 1 (Bounded localization uncertainty): There exist known constants $0 < \underline{c} \leq \overline{c} < \infty$ such that, for every agent i and all times t,

$$\underline{c} \;\leq\; \lambda\big(C_i(t)\big) \;\leq\; \overline{c} \tag{12}$$

for every eigenvalue $\lambda\big(C_i(t)\big)$ of the localization covariance matrix $C_i(t)$. In other words, all eigenvalues of $C_i(t)$ lie in the interval $[\underline{c}, \overline{c}]$, so the covariance is uniformly bounded and nondegenerate.

Assumption 2 (Safe rendezvous feasibility): There exists $\varepsilon > 0$ such that, for all $(i,j) \in \mathcal{E}$,

$$W_2\big(\mu_i(0), \mu_j(0)\big) \;\geq\; d_{ij}^{\text{eff}} + \varepsilon, \tag{13}$$

so that the initial configuration lies strictly inside the safe set $\mathcal{C} = \{\, W_2(\mu_i, \mu_j) \geq d_{ij}^{\text{eff}} \,\}$, and all Wasserstein-based weights $w_{ij}(t)$ remain well-defined on the corresponding forward-invariant region. Moreover, the desired rendezvous or formation configuration is assumed to be *compatible with safety*, in the sense that it lies in the closure of $\mathcal{C}$, so that the CBF–QP constraints remain feasible along the trajectories.

Theorem 1: Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a connected, undirected graph of N agents, and suppose Assumption 1 holds so that each agent maintains a Gaussian belief $\mu_i(t) = \mathcal{N}(m_i(t), C_i(t))$. Let $w_{ij}(t)$ and the nominal consensus dynamics be defined as in (5)–(6).

(i) Nominal consensus. Assume $\|m_i(0) - m_j(0)\| \geq d_{ij}^{\text{eff}} + \varepsilon$ for all $(i,j) \in \mathcal{E}$ and some $\varepsilon > 0$, and that Assumption 2 holds. Then under (5) all means converge exponentially to $m_{\text{avg}} = \frac{1}{N}\sum_{k=1}^{N} m_k(0)$, and the disagreement decays with a rate lower bounded by $\lambda_2(\mathcal{L})/(1+\delta)$, where L is the static distance-weighted Laplacian.

(ii) Safe consensus with CBF–QP. Let h_{ij}, d_{ij}^{eff}, and the safe set $\mathcal{C} = \{m : h_{ij} \geq 0,\ \forall (i,j) \in \mathcal{E}\}$ be as in (9). Let u_i^{nom} be given by (5), and let $u^\star(t)$ be the solution of the CBF–QP (11). If $m(0) \in \mathcal{C}$ and the CBF constraints are feasible on $\mathcal{C}$, then $\mathcal{C}$ is forward invariant and the closed-loop trajectories converge to a compact d_{ij}^{eff}-safe consensus set in which the means are aligned to enforce the safety margin.

Proof: We prove the two parts separately.

(i) Nominal exponential consensus. For Gaussian beliefs $\mu_i = \mathcal{N}(m_i, C_i)$ and $\mu_j = \mathcal{N}(m_j, C_j)$, the squared 2-Wasserstein distance satisfies

$$\|m_i - m_j\| \leq W_2(\mu_i, \mu_j) \leq \sqrt{\|m_i - m_j\|^2 + c_\delta}, \tag{14}$$

since $\lambda\big(C_i(t)\big)\,\lambda\big(C_j(t)\big) \;\leq \delta$. Hence the weights $w_{ij} = 1/(W_2 - R)$ remain positive and bounded, implying that $\mathcal{L}(t)$ is a symmetric Laplacian with bounded weights.

Let $m_{\text{avg}} = \frac{1}{N}\sum_{k=1}^{N} m_k(0)$ and $V = \frac{1}{2}\sum_{i=1}^{N} \|m_i - m_{\text{avg}}\|^2$. Since the nominal dynamics are Laplacian, m_{avg} is invariant and

$$\dot{V} = -X^\top \mathcal{L}(t) X \leq -2\lambda_2(\mathcal{L}(t))V. \tag{15}$$

Using bounded covariance, $\mathcal{L}(t) = \mathcal{L} + \Delta\mathcal{L}(t)$ with $\|\Delta\mathcal{L}(t)\| \leq c\delta$, and Weyl's inequality gives

$$\lambda_2(\mathcal{L}(t)) \geq \frac{\lambda_2(\mathcal{L})}{1+\delta}. \tag{16}$$

Thus,

$$V(t) \leq V(0)\exp\!\Big(-\tfrac{\lambda_2(\mathcal{L})}{1+\delta}t\Big), \tag{17}$$

which proves exponential consensus.

(ii) Chance-constrained safety under the CBF-QP. Define $h_{ij} = \|m_i - m_j\| - d_{ij}^{\text{eff}}$. Its derivative is affine in the control inputs, and since the cost is quadratic, the QP is convex with a unique solution. If $m(0) \in \mathcal{C}$ and the CBF condition $\dot{h}_{ij} \geq -\alpha(h_{ij})$ holds, the comparison lemma guarantees forward invariance of $\mathcal{C}$.

Because $p_i - p_j \sim \mathcal{N}(m_i - m_j, C_i + C_j)$, the inflated margin d_{ij}^{eff} ensures

$$\mathbb{P}(\|p_i - p_j\| \geq d_{ij}^{\text{eff}}) \geq 1 - \varepsilon, \tag{18}$$

establishing probabilistic safety. The QP returns the closest feasible input to the nominal Wasserstein consensus, recovering exponential convergence when constraints are inactive. Otherwise, trajectories remain in the invariant safe set, and by LaSalle's principle, the system converges to a d_{ij}^{eff}-safe consensus set. ■

C. Formation Control

We now consider formation control under the same Gaussian localization model $\mu_i = \mathcal{N}(m_i, C_i)$. The goal is to maintain a desired formation defined by relative distance constraints $d_{ij}^{\text{des}} > 0$ for each $(i,j) \in \mathcal{E}$:

$$\|m_i - m_j\| = d_{ij}^{\text{des}}, \quad \forall (i,j) \in \mathcal{E}. \tag{19}$$

Due to localization uncertainty, we express the formation constraint using the Wasserstein distance between the Gaussian beliefs μ_i and μ_j. The distance $W_2(\mu_i, \mu_j)$ is given by the expression above and accounts for both mean separation and covariance differences, making the formation control robust to sensor noise. We propose the distributed formation controller

$$u_i = \dot{m}_i = -\sum_{j \in \mathcal{N}_i} w_{ij} \left(W_2(\mu_i, \mu_j) - d_{ij}^{\text{des}} \right) \frac{m_i - m_j}{W_2(\mu_i, \mu_j)}, \tag{20}$$

with

$$w_{ij} = \frac{1}{W_2(\mu_i, \mu_j) - d_{ij}^{\text{th}}}, \qquad (i,j) \in \mathcal{E}. \tag{21}$$

Here, $W_2(\mu_i, \mu_j)$ replaces the standard Euclidean distance in both the direction and the gain of the controller, explicitly incorporating localization uncertainty.

Theorem 2: Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a connected, undirected graph of N agents, and suppose Assumption 1 holds so that each agent maintains a Gaussian belief $\mu_i(t) = \mathcal{N}(m_i(t), C_i(t))$. For each $(i,j) \in \mathcal{E}$, let $d_{ij}^{\text{des}} > 0$ be the desired distance, and let the Wasserstein-based formation controller $u_i^{\text{nom}}(t)$ and weights $w_{ij}(t)$ be given by (20)–(21).

(i) Nominal formation. Assume Assumption 2 holds and that $\{d_{ij}^{\text{des}}\}$ define a rigid formation on $\mathcal{G}$. Then the dynamics (20) ensure exponential convergence of all edge errors $e_{ij}(t) = \|m_i(t) - m_j(t)\| - d_{ij}^{\text{des}}$, with disagreement decaying at a rate lower bounded by $\lambda_2(\mathcal{L})/(1+\delta)$, where L is the static Euclidean-distance Laplacian.

(ii) Safe formation with CBF–QP. Let h_{ij}, d_{ij}^{eff}, and the safe set $\mathcal{C} = \{m : h_{ij} \geq 0, \ \forall (i,j)\}$ be defined as in (9). Assume $d_{ij}^{\text{des}} \geq d_{ij}^{\text{eff}}$ and that the CBF constraints in the CBF–QP are feasible for all $m \in \mathcal{C}$. Let u_i^{nom} be given by (20), and let $u^\star(t)$ be the solution of the CBF–QP (11) with an extended class-$\mathcal{K}$ function α. If $m(0) \in \mathcal{C}$, then $\mathcal{C}$ is forward invariant, and the closed-loop trajectories converge to a compact d_{ij}^{eff}-safe formation set in which the inter-robot distances achieve the desired formation up to the enforced safety margin.

Proof: The proof is similar to Theorem 1 and is therefore omitted here. ■

Remark 1: Each robot uses a range sensor to measure relative distances to neighbors within a fixed sensing radius. Measurements are modeled as Gaussian beliefs $\mu_i(t) = \mathcal{N}(m_i(t), C_i(t))$, where $C_i(t)$ is a time-varying covariance satisfying Assumption 1. The consensus weights w_{ij} depend nonlinearly on the means and covariances, so the closed-loop joint distribution of the agents' states is not guaranteed to remain Gaussian. Assumptions 1 and 2 ensure that the interaction weights are bounded and smooth, so the mean dynamics are well defined and suitable for stability analysis. When covariances are small and the weights vary slowly, the Gaussian approximation remains accurate, and the mean dynamics provide a reliable description of the posterior evolution.

In practice, slowly varying biases (e.g., drift or miscalibration) appear as systematic shifts in m_i: the Wasserstein controller still achieves consensus in belief space, but the consensus point may be offset in the global frame, while relative objectives (rendezvous, formation) are preserved. Since the analysis only requires finite second moments, non-Gaussian or heavy-tailed noise appears through inflated covariances, which naturally reduce the influence of highly uncertain neighbors. However, under-reported covariances may over-attract faulty or biased agents; Assumption 1 rules out degenerate covariances. Extending the framework to fully non-Gaussian posteriors and explicitly fault-tolerant Wasserstein consensus schemes is left for future work.

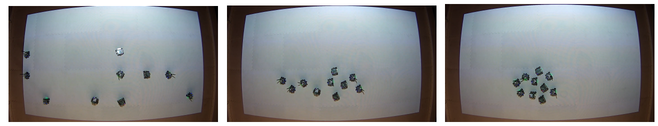
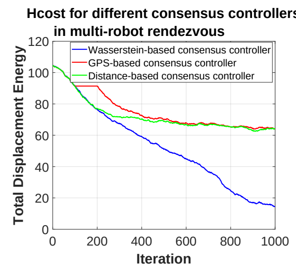
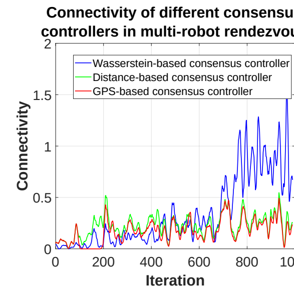


Fig. 2: Illustration of rendezvous task experiments conducted in the Robotarium hardware platform [14], showing the evolution of robot trajectories over time.



(a) Hcost (displacement energy over time) for different consensus controllers in distributed multi-robot rendezvous



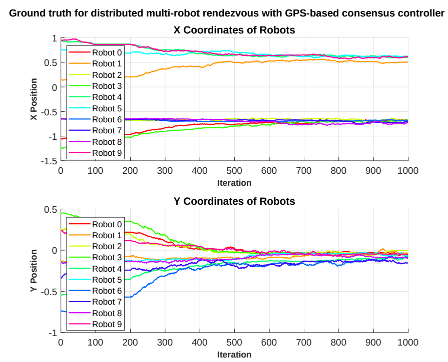
(b) Connectivity of different consensus controllers in distributed multi-robot rendezvous

Fig. 3: The total displacement energy (Hcost) for different consensus controllers in distributed multi-robot rendezvous under various scenarios.

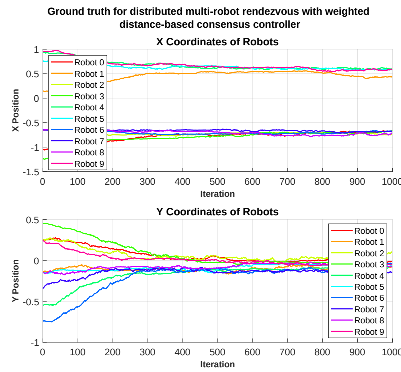
Remark 2: The Wasserstein-based controllers are derived for single-integrator dynamics $\dot{m}_i = u_i \in \mathbb{R}^2$, while the robots follow unicycle kinematics. We interpret $u_i = [u_{x,i}, \, u_{y,i}]^\top$ as a desired planar velocity and implement $v_i = u_{x,i} \cos\theta_i + u_{y,i} \sin\theta_i$, $\omega_i = k_\omega \left(-u_{x,i} \sin\theta_i + u_{y,i} \cos\theta_i \right)$ with $k_\omega > 0$ and saturated v_i. Since the term scaled by k_ω penalizes heading misalignment, choosing k_ω sufficiently large yields fast alignment and keeps the unicycle trajectories close to the single-integrator behavior assumed in the analysis.

V. Experiments

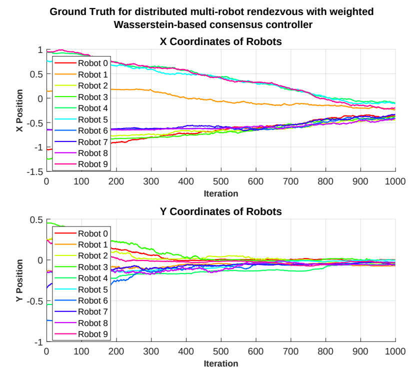
The simulations and hardware experiments conducted on the Robotarium platform [14] provide a detailed evaluation of various consensus controllers for distributed multi-robot rendezvous and circular formation tasks. The experimental setup consists of $N = 10$ robots, initially placed randomly in the 3.2 × 2 meter rectangular workspace, each following a distinct consensus strategy to achieve rendezvous and formation shape. For benchmarking, we compare the proposed *Wasserstein-based consensus controller* with several baseline methods, including the *distance-based consensus controller* [19] and the *GPS-based consensus controller* [24]. The robots operate under Gaussian localization uncertainty, where the covariance varies randomly at each iteration, leading to time-varying but bounded uncertainty. All controllers are implemented using the OSQP solver to ensure a fair and consistent comparison across methods.



(a) GPS-based consensus controller

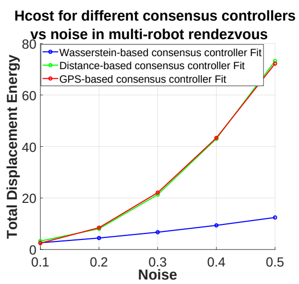


(b) Weighted distance-based consensus controller

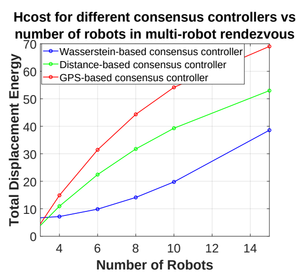


(c) Weighted Wasserstein-based consensus controller

Fig. 4: Ground truth trajectories of the distributed multi-robot rendezvous with different consensus controllers.



(a) Hcost (total displacement energy) for different consensus controllers vs noise in distributed multi-robot rendezvous.



(b) Hcost (total displacement energy) for different consensus controllers vs the number of robots in distributed multi-robot rendezvous example.

Fig. 5: Impact of the uncertainty and connectivity on the Rendezvous task.

A. Rendezvous Task

The experimental snapshots, as depicted in Fig. 2, demonstrate that the proposed *Wasserstein-based consensus controller* effectively minimizes deviation and achieves consensus (rendezvous) among the robots, even under the imposed localization uncertainty constraints. To assess the performance of each approach, we employ the *total displacement energy* (Hcost) as a metric, serving as a Lyapunov candidate function that evaluates the collective deviation of robots from a common rendezvous point. The total displacement energy is formally defined as

$$Hcost = V_R = \sum_{i \in V} \sum_{j \in V} a_{ij} \|p_j - p_i\|, \tag{22}$$

where p_i and p_j denote the current positions of robots i and j, respectively, N_i represents the set of neighbors of robot i, and a_{ij} is the entry of the adjacency matrix that quantifies the connectivity strength between the robots. A decrease in V over time indicates successful convergence, with a steeper decline corresponding to a faster achievement of consensus.

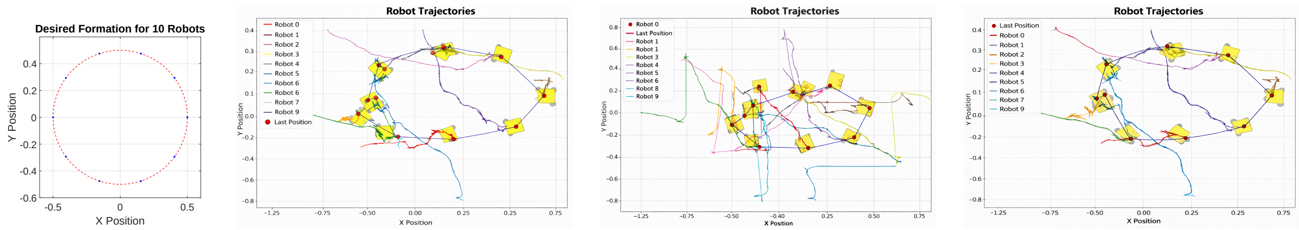
Fig. 3(a) illustrates that the proposed *Wasserstein-based consensus controller* achieves the most efficient reduction in displacement energy, reaching a stable state significantly faster than the other approaches. The *distance-based* and *GPS-based* consensus controllers exhibit similar convergence behavior in this scenario, with comparable residual displacement energy. This indicates that, under low uncertainty, both relative (distance-based) and absolute (GPS-like) sensing lead to comparable performance. The advantage of the Wasserstein-based design becomes more pronounced in higher-uncertainty settings, where it explicitly incorporates localization uncertainty into the consensus process.

Fig. 3(b) also shows the connectivity of robots during the distributed rendezvous process with different controllers. The limited sensing range affects the effectiveness of the controllers, as connectivity between robots dynamically changes based on their relative positioning. Notably, the Wasserstein-based consensus controller demonstrates resilience to these constraints, leveraging optimal performance to ensure robust coordination despite localization noise.

Fig. 4 presents the ground truth trajectory data for the robots' coordinates, which further validate the performance differences among the controllers. The GPS-based consensus controller exhibits noticeable deviations from the ideal trajectories as localization inaccuracies propagate through the consensus process. Although the distance-based controller eventually converges, it demonstrates minor overshoot and oscillatory behavior—particularly in the Y-coordinate trajectories. In contrast, the Wasserstein-based consensus controller exhibits smooth trajectory evolution, ensuring minimal deviation and maintaining stability throughout the rendezvous process.

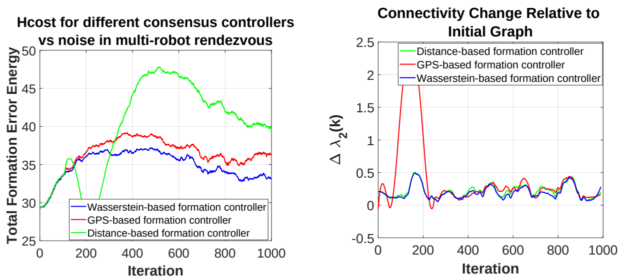
Furthermore, as illustrated in Fig. 5(a), the Wasserstein-based consensus controller demonstrates superior robustness to noise when compared with the distance-based and GPS-based controllers. The total displacement energy for the Wasserstein-based approach remains relatively low across a wide range of noise levels, whereas both the GPS-based controller and the distance-based controllers exhibit exponentially scaling sensitivity to noise. This behavior suggests that the state-of-the-art Euclidean distance-based method is more susceptible to localization errors, ultimately leading to inefficient movements.

These findings highlight the advantage of employing the Wasserstein-based method to ensure stable and energy-efficient rendezvous, even at significant noise levels. The significance can be generalized to a broader class of multi-robot coordination problems, such as flocking and swarm intelligence, where consensus protocols are key. Moreover, the scalability analysis presented in Fig. 5(b) indicates that as the number of robots increases, all controllers experience a rise in total displacement energy, with the GPS-based con-



(a) Desired Formation (b) Distance-based formation controller (c) GPS-based formation controller (d) Wasserstein-based formation controller

Fig. 6: Trajectories of the robots and their final positions during the circle formation task with different consensus controllers with localization uncertainty.



(a) Fcost for different formation controllers in distributed multi-robot formation. (b) Connectivity of different formation controllers in distributed multi-robot formation.

Fig. 7: Formation energy cost (Fcost) and the connectivity evolution during the circle formation task with the consideration of localization uncertainty.

troller exhibiting the highest cost. The Wasserstein-based approach consistently maintains the lowest energy consumption across varying team sizes, demonstrating its effectiveness in large-scale MRS. Although the distance-based controller outperforms the GPS-based method, it still incurs higher displacement energy than the Wasserstein-based counterpart. These results highlight that the Wasserstein-based consensus controller not only enhances robustness against noise but also scales efficiently, making it a promising solution for distributed robotic applications requiring energy-efficient coordination and adaptability in uncertain environments.

B. Circle Formation

The experimental results (Fig. 6) demonstrate the effectiveness of the proposed Wasserstein-based consensus controller in achieving robust circle formation, which is generally considered a difficult-to-achieve formation shape due to its non-rigid connections. The performance is evaluated using the total formation error energy as $V_F = \sum_{i\in V}\sum_{j\in V} a_{ij} \left\| \|p_j - p_i\| - d_{ij}^{\mathrm{des}} \right\|$ which quantifies the deviation from the desired formation geometry. As shown in Fig. 7(a), while the Wasserstein-based controller initially exhibits higher displacement energy, it achieves a lower steady-state error compared to the GPS-based and distance-based controllers, indicating improved convergence. Fig. 7(b) shows the relative connectivity, defined as the change in algebraic connectivity with respect to the initial graph. The proposed Wasserstein-based method exhibits smaller variations, indicating greater robustness to noise-induced disruptions. In contrast, the baseline methods show larger fluctuations due to noisier robot motions.

The trajectory plots in Fig. 8 further confirm that our approach facilitates a more structured and coordinated formation, with smoother adjustments in both x-y coordinates. The consistent reduction in V over time validates the superior resilience and adaptability of the Wasserstein-based controller, making it a more effective solution for distributed multi-robot formation control.

C. Consensus with Application to Target Detection

In this experiment, a team of robots is tasked with moving toward a common target while navigating a cluttered environment populated with static obstacles. The robots must simultaneously avoid collisions, preserve network connectivity, and achieve a desired goal region, all under localization uncertainty. By explicitly incorporating the localization uncertainty into the Wasserstein-based controller, our method adapts the interaction weights, which improves safety near obstacles and yields more reliable convergence to the target.

Fig. 9 presents the experimental results, which confirm the effectiveness of the proposed framework. Fig. 9(a) reports the target detection performance of three approaches, where the Wasserstein-based controller achieves faster and more consistent target detection. Figs. 9(b)–(d) show the corresponding robot trajectories, demonstrating that the proposed method ensures safe navigation toward targets while maintaining inter-agent connectivity and avoiding static obstacles. Collectively, these results highlight our framework's capability to provide a robust and unified solution for adaptive multi-robot coordination with localization uncertainty.

VI. Conclusion

This paper presented a Wasserstein-based consensus and formation control framework for multi-robot coordination that explicitly accounts for localization uncertainty. By modeling robot positions as Gaussian beliefs and leveraging the Wasserstein distance in the coordination layer, the proposed approach improves robustness to sensor noise and communication constraints. In addition, a chance-constrained CBF–QP safety layer was integrated to ensure probabilistic collision avoidance and connectivity preservation under uncertain localization. A convergence analysis established a lower bound on the rate as a function of uncertainty and network topology. Extensive experiments demonstrated that the proposed method outperforms conventional distance-based and GPS-dependent controllers in both consensus and formation tasks. The framework achieves faster convergence, improved formation stability, and enhanced robustness to localization noise while maintaining safety.

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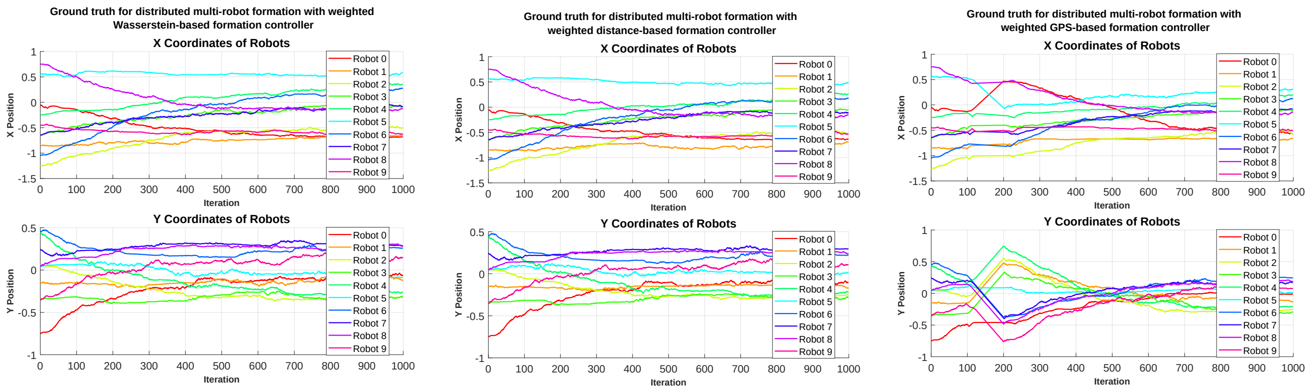
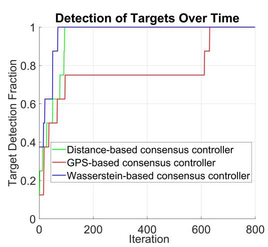


(a) Ground truth for distributed multi-robot formation with Wasserstein-based formation controller

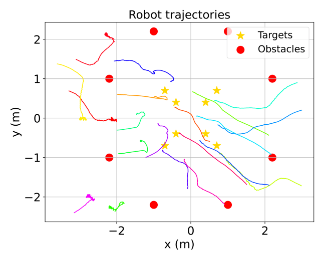
(b) Ground truth for distributed multi-robot formation with weighted distance-based formation controller

(c) Ground Truth for distributed multi-robot formation with weighted GPS-based formation controller

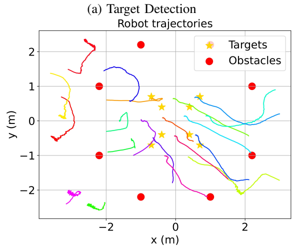
Fig. 8: Ground truth trajectories of the robots during the distributed multi-robot formation with different formation controllers.



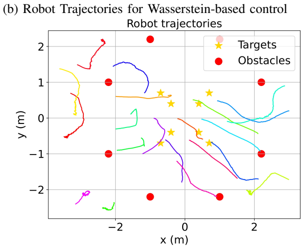
(a) Target Detection



(b) Robot Trajectories for Wasserstein-based control



(c) Robot Trajectories for distance-based control



(d) Robot Trajectories for GPS-based control

Fig. 9: Performance evaluation of the proposed multi-robot framework in a scaled target detection scenario. (a) Target detection results for three approaches. (b)–(d) Robot trajectories toward targets while avoiding obstacles under the three approaches.

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